

**PARAMETERS OF DISTRIBUTION OF THE PRIMARY
GAMMA-TRANSITION INTENSITIES FOLLOWING RESONANCE
NEUTRON CAPTURE AND SOME PROPERTIES OF COMPOUND
NUCLEI $^{157,159}\text{Gd}$**

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Abstract

The re-analysis of the published experimental data on the primary gamma-transition intensities following neutron capture in different groups of neutron resonances in $^{156,158}\text{Gd}$ has been performed. There are determined the most probable values of sum of E1 and M1 primary transitions, numbers of excited by them levels of both parities, ratios of radiative strength functions $k(\text{M1})/k(\text{E1})$, dispersions of deviations of random values of intensities from the average and ratios of mean intensities of primary transitions to levels $J=5/2$ with respect to analogous data for $J=1/2$ and $3/2$ (capture of the 24 keV neutrons) in narrow excitation energy intervals.

All the data on level density and sums of radiative strength functions confirm the presence of clearly expressed step-like structure in level density below 3 MeV and general trend in change in strength functions as changing primary gamma-transition energy. Variations of distribution dispersions and, especially, ratio $k(\text{M1})/k(\text{E1})$ (or $k(\text{E1})/k(\text{M1})$) at changing excitation energy point to strong change in structure of these nuclei above 1.0-1.5 MeV.

As in neighboring isotopes $^{156,158}\text{Gd}$, the shape of energy dependence of $k(\text{M1}) + k(\text{E1})$ considerably differs as changing nuclear mass. This can be due to dependence of gamma-decay process on structure of neutron resonance and/or levels excited by gamma-transitions. The dilemma appeared can be solved only in direct experimental search for structure of neutron resonances in region of their energy of about two nucleon pairing energy in nuclei of corresponding mass.

1 Introduction

Density of excited levels ρ and emission probability Γ of any nuclear reaction product following neutron resonance λ decay are the main sources of experimental information on properties of nuclear matter and practically important nuclear-physics constants. This stipulates for necessity of their determination with maximum possible accuracy.

However, the ρ and Γ values cannot be determined for main mass of nuclei excited in (n, γ) reaction in direct experiments: mean spacing D between levels is comparable with or much less than a resolution (FWHM) of existing spectrometers. Correspondingly, these parameters can be extracted only from the spectra measured with “bad” resolution. Only one-step reactions were mainly used for this aim. Obtaining of information on ρ and Γ from gamma-spectra of two-step reaction was started only in the last time [1]. Comparison of shape of functional dependence of the obtained in this way values of $\rho = f(E)$ and radiative strength functions $k = \Gamma_\lambda/(E_\gamma^3 D_\lambda A^{2/3}) = \phi(E_\gamma)$ with the data of one-step reactions (spectra of evaporated nucleons, different gamma-spectra) points to

their principle incompatibility. It appears itself in presence [1] or lack of abrupt changes in determined parameters. This operation allows one to determine sources of systematical errors, estimate their values and reveal the region of maximum discrepancy between the data of different experiments.

Analysis of the most sufficient sources of systematical errors [2] and their transfer coefficients onto errors of parameters in practically realized case of two-step reaction (two simultaneously emitted gamma-quanta) showed that even maximum possible errors of the ρ and Γ values (obtained from two-step reaction) cannot explain discrepancy for one- and two-step reactions.

Nevertheless, necessity of additional testing a method for determination of ρ and Γ from two-step reactions calls no doubts. At present, such test can be performed only in model-less analysis of the primary gamma-transition intensities from reaction (\bar{n}, γ) (capture in “averaged” resonances).

Possibility to obtain new information from these data is caused by the use by authors of experiments of unnecessary for analysis ideas of “statistical” mechanism of gamma-decay process and abstract form of distribution law of arbitrary gamma-transition intensity deviation in individual resonance from mean value. The shape of approximating function and concrete results of analysis of rather limited set of strongly fluctuating data are given in [3]. More general variant of analysis is described in [4].

2 Experimental data

In the isotopes under consideration $^{157,159}\text{Gd}$ were measured the primary transition intensities following resonance neutron capture with mean energies of 2 and 24 keV; there are also data for ^{159}Gd for both capture of neutrons in resolved resonances and on beam with cadmium filter. Experimental data cover maximum level excitation energy diapason for even-odd deformed nuclei and allow one to get maximum possible and completely independent information on the desired gamma-decay parameters.

In two last cases s-neutrons are mainly captured, but capture of p-neutrons must be taken into account at $E_n=2$ and 24 keV. According to [5], ratio between available in that time values of strength functions of s- and p-neutrons does not exceed 0.08 for $E_n=2$, but is grater than 1 for 24 keV. Therefore, the following analysis should account for excitation of resonances with spins $1/2, 3/2$ and different parity only for the data $E_n=24$ keV. This means that the averaged intensity of the primary dipole gamma-transitions to levels $5/2$ depends on ratio between strength functions S_0, S_1 and that it must be varied at determination [4] of distribution parameters of the random intensity deviations from mean value. In principle, level density with spin $5/2$ in deformed nuclei must be bigger than corresponding sum for spins $1/2, 3/2$ (as it approximately follows from known functional dependences $\rho = f(J)$). In case of lower neutron energy, there was adopted hypothesis of equal intensity of primary gamma-transitions with near energy and the same multipolarity to levels with spins $1/2, 3/2$.

This assumption can be mistaken for strongly differing structure of wave functions of excited levels with different spin. According to modern microscopic nuclear models, amplitude of gamma-transition is determined by a set of quasiparticle and phonon components. Their contribution depends on wave functions of both decaying and excited levels (simplified expression for matrix element of gamma-transition in even-odd nucleus

is given for example, in [6]).

3 Required parameters of analysis

Examples of random intensity distributions in integral form for their different N_γ values in given level excitation energy interval, ratios $k(M1)/k(E1)$ of strength functions of E1- and M1-transitions, dispersions of distributions $\sigma^2 = 2/\nu$ and registration threshold of peak in spectrum are given in [4] for the case of excitation of resonances with the only spin value.

Introduction of additional parameter (ratio between mean intensities of primary gamma-transitions to levels 5/2 and corresponding data for levels 1/2, 3/2) in the case of resonance with two spin values weekly influences stability of approximation and sensitivity of this process to variation of initial values of fitted parameters. This result was tested for the case when mean intensities in two different groups differed, practically, by a factor 2.

4 Results of analysis

Experimental cumulative sums of relative intensities $\langle I_\gamma/E_\gamma^3 \rangle$ are presented in figs. 1,2 together with their best approximations. As an example, there are used the data for $E_n = 2$ keV. Due to negligible contribution of p-neutron capture, approximation takes into account only two distributions – E1- and M1-transitions following decay of resonances with spins 1/2. Experimental distributions for $E_n = 24$ keV are superposition of four distributions – two distributions mentioned above and one more pair corresponding to gamma-transitions between resonances with $J=3/2$ and final levels with $J=5/2$. Its mean intensity relatively to gamma-transition intensities to levels $J=1/2, 3/2$ is found equal ≈ 0.44 for both isotopes (but with some larger dispersion of values for ^{159}Gd).

All the data are presented so that the expected total intensity of gamma-transitions lying below registration threshold corresponds to the most probable value of cumulative sum for zero intensity.

Precision in determination of parameters of approximating curve at low energy of final levels E_i must get worse due to inequality of level densities with different parity. Most probably, this increases error of extrapolation by approximating curve to zero intensity of gamma-transition. In practice, overestimation of the N_γ values seems to be more probable.

The best values of fitting parameters $R_k = k(M1)/k(E1)$ and ν are presented in figs. 3 and 4. Main part of data in fig. 3 corresponds to the case $k(M1)/k(E1)$. In potentially possible case $k(E1) \leq k(M1)$, some portion of the data shown in these figures corresponds to alternatively determined ratio $k(E1)/k(M1)$. These cases cannot be revealed without the use of additional experimental information. Noticeable change in $R_k = k(M1)/k(E1)$ and ν at ≈ 1 MeV points to considerable change in structure of isotopes under study at this excitation energy. As it is seen from figure 4, fluctuations of random intensities to final levels of ^{159}Gd with spins 1/2, 3/2, from the one hand, and 5/2, from the other hand, are described by distributions with rather different values of ν . This fact has, in

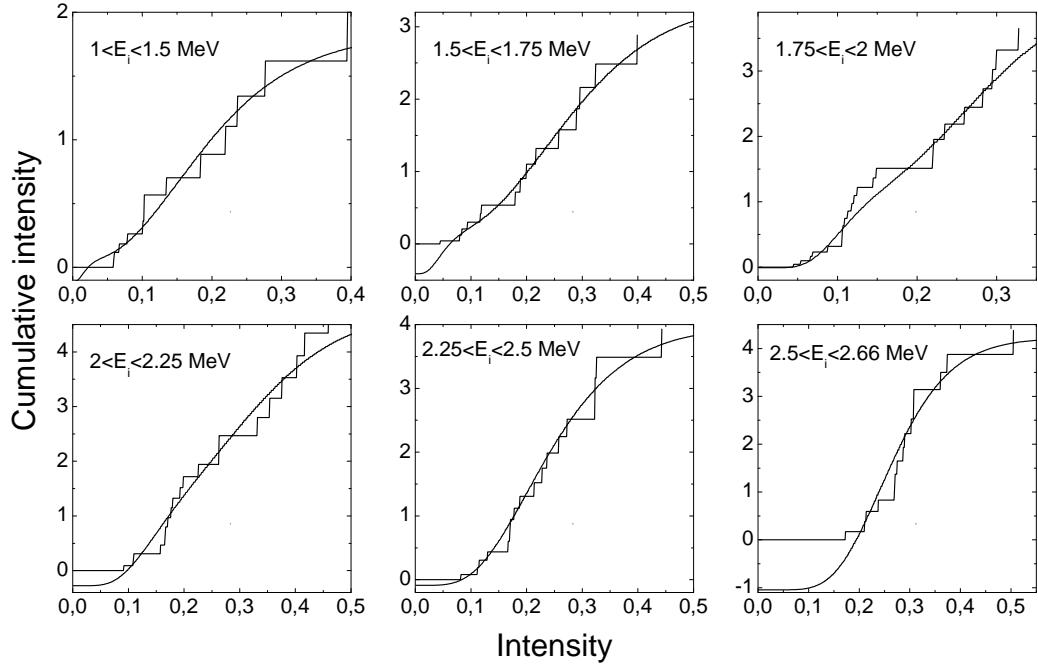


Fig. 1. The experimental cumulative sum of reduced intensities $\langle I_\gamma / E_\gamma^3 \rangle$ for ^{157}Gd - histogram. Smooth curve corresponds to the best approximation. Excitation energy intervals of cascade final levels E_i are given in figure. Experimental data for neutron energy $\approx 2 \text{ keV}$.

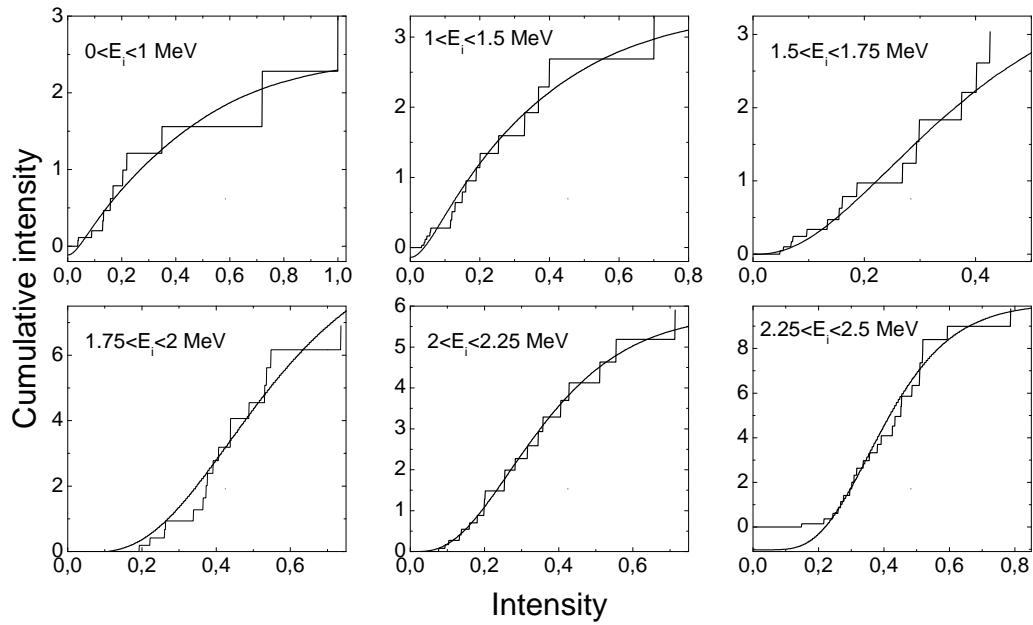


Fig. 2. The same, as in Fig. 1, for ^{159}Gd .

principle, the following interpretation:

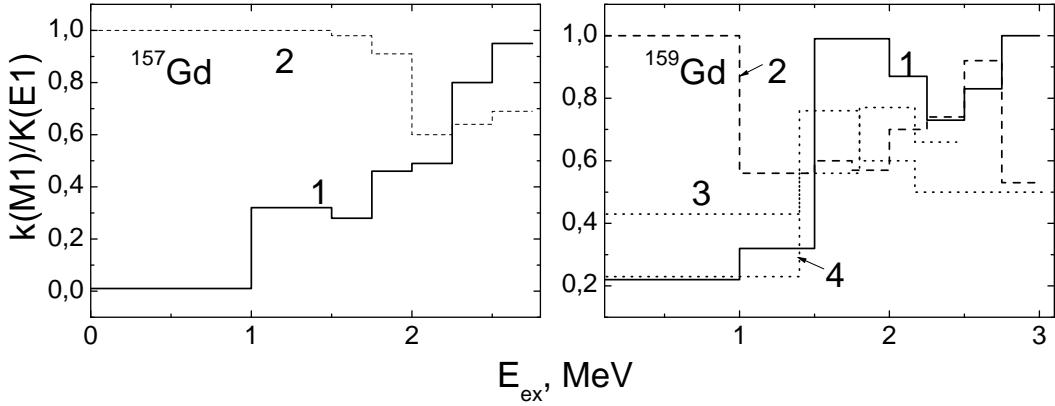


Fig. 3. The best values of ratios $k(\text{M1})/k(\text{E1})$ (or $k(\text{E1})/k(\text{M1})$) for different energy of levels excited by dipole gamma-transitions in $^{157,159}\text{Gd}$. Line 1 represents data for $E_n \approx 2$ keV, line 2 - for $E_n \approx 24$ keV. Line 3 – data for $E_n > 1$ eV, line 4 – data for isolated resonances

- (a) distinction in structure of decaying neutron resonances with different spin and parity;
- (b) amplitude of corresponding gamma-transitions is determined by components of wave functions which differ (see, for example, [9]) in number of phonons and degree of their fragmentation;
- (c) different structure and degree of fragmentation of the levels excited by primary gamma-transitions.

In principle, one cannot exclude possibility of presence of some specific systematical uncertainty which explains this effect. But, there is required realistic explanation for its selectivity with respect to resonance spins. Unreality of existence of assumed uncertainty brings to considerable conclusion: there are not grounds to exclude possible appearance of the different ν values for gamma-transitions with different multipolarity to final levels

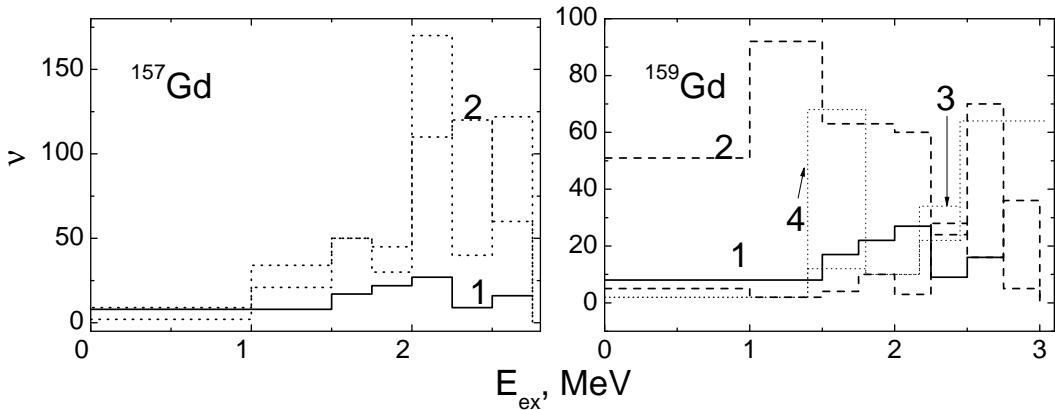


Fig. 4. The same, as in Fig. 3, for parameter ν for $^{157,159}\text{Gd}$.

with different spin. The data in figs. 3,4 unambiguously point to evident change in structure of wave functions of final levels below and above ≈ 1 MeV.

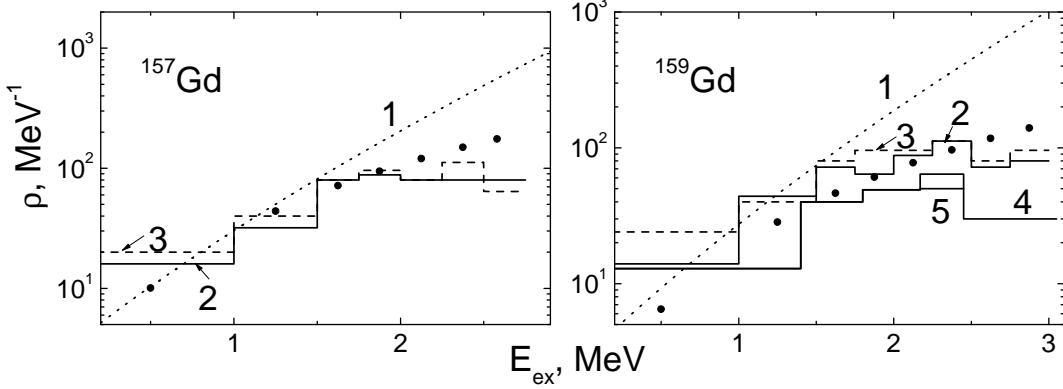


Fig. 5. Different data on level density in ^{157,159}Gd. Dotted curves 1 represent results of calculation within model [16]. From the data for $E_n \approx 2$ – histogram 2; $E_n \approx 24$ keV – histogram 3. Points - the best fit of data for $E_n \approx 2$ within model [14] for $K_{coll} = \text{const}$. Histogram 4 shows data for $E_n > 1$ eV, histogram 5 - for isolated resonances.

The best values of level density $\rho = \sum_{J,\pi} N_\gamma / \Delta E$ and sums of radiative strength functions $\sum < I_\gamma > / (E_\gamma^3 N_\gamma)$ are shown in figs. 5, 6. Normalization of intensities and strength functions in both [7] and [8] was performed for five (from six) data sets.

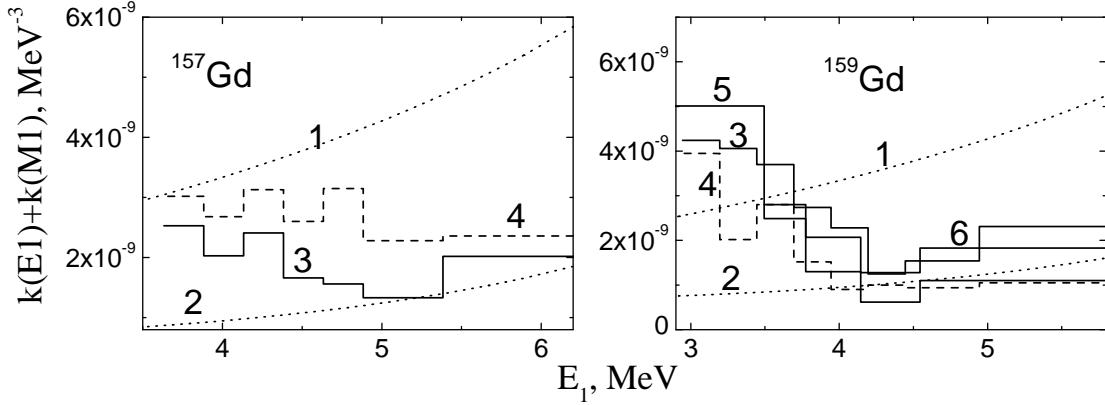


Fig. 6. Different data on radiative strength functions of the primary gamma-transitions in ^{157,159}Gd. Dotted curve 1 shows results of calculation according to model [17]; dotted curve 2 – calculation within model [18] in sum with $k(M1) = \text{const}$. From the data for $E_n \approx 2$ – histogram 3; $E_n \approx 24$ keV – histogram 4. From the data for $E_n > 1$ eV – histogram 5; isolated resonances – histogram 6.

Because intensities of gamma-transitions following “filtered” neutron capture are given in [7, 8] in relative units then corresponding strength functions in figs. 5, 6 are combined with each other under assumption of their approximate equality for final levels $E_i < 1$ MeV.

The most probable approximated N_γ values from capture spectra of 2 keV and 24 keV neutrons in ^{158}Gd below 1.0 and above 2.5 MeV exceed “resonance” values, practically, by a factor of 2 (and some less - in other excitation energy intervals).

4.1 Some sources of systematical errors

Absolute minimum of χ^2 for all the used sets of intensities is achieved, practically, for the only value of N_γ , if the value of this variable equals ~ 5 . Change in this parameter by ± 1 brings usually to considerable increase in χ^2 .

This allows one not to account for possibility of considerable (for example, more than 10-20%) uncertainty in determined level density caused by chosen conditions of approximation (excluded inequality of level densities with different parity).

Main problems in determination of nuclear parameters and their systematical errors are, most probably, caused by:

- (a) the use of assumptions on shape of the random intensity deviations from mean value and
- (b) possible presence of significant systematical errors in sets of analyzed intensities [7, 8].

1. The Porter-Thomas distribution allows very significant random partial widths. But fluctuations of the measured gamma-transition intensities $I = \Gamma_{\lambda i}/\Gamma_\lambda$ are always limited in their maximum value by positive correlation between partial and total radiative widths of decaying level. This results in some overestimation of number of degrees of freedom ν determined by approximation and its dependence on intensities included in approximation of cumulative sums. For instance, there is region of values $I_\gamma/(E_\gamma^3) > 0.4$ for $1 < E_i < 1.5$ MeV shown in Fig. 1. Intensities in the performed analysis were normalized so that their maximum value did not exceed $\sim 50\%$ of approximation region for the majority of data sets.

2. The main error of analysis can be related only to “loss” of gamma-transitions whose intensities do not exceed threshold value and/or mistaken identification of quanta ordering in gamma-cascades.

Probability of overlap of two peaks corresponding to near-lying levels was estimated in [10]. As it follows from the data presented by authors, this effect is small enough and, most probably, cannot explain significant (several times) discrepancy between level density determined by us and its prediction in the frameworks of the Fermi-gas model. Moreover, this overlapping in the chosen presentation of experimental data increases rate of growth of cumulative sums and, most probably, overestimates the N_γ value obtained. There could be essential uncertainty caused by even and considerable loss of some part of observed peaks corresponding to intense primary transitions due to groping of levels in near-lying (spacing of about 1-2 keV) multiplets. But this possibility is not predicted by modern nuclear theory. Experimental data of nuclear spectroscopy also do not point to existence of numerous multiplets of neighboring levels with spins $1/2$ and $3/2$ in even-odd compound nuclei.

3. It is also possible that the gamma-transitions in the all or greater part of chosen intervals of primary transition intensities (with the width of some hundreds keV) have different mean values. Moreover, probability of relatively low-intensity gamma-transitions

quickly (but smoothly) increases as decreasing their intensity. In principle, this effect can be caused by mechanism of fragmentation [9] of different states over neighboring levels of a nucleus.

Apparently, only this hypothesis can be alternative explanation of “step-like” structure in level density in performed here analysis. This hypothesis can be applied to the determined according to [1, 11] level densities under the following conditions: main part of levels (with the same J^π) below $\sim 0.5B_n$ must not be excited by primary gamma-transitions; some Cooper pairs of nucleons must break simultaneously at small as compared with B_n nuclear excitation energy. One cannot suggest other possibility for precise calculation of the two-step gamma-cascade intensities in the investigated even-odd nuclei.

4.2 Interpretation of the obtained results

The most important physics information on structure of excited levels below $\approx 0.5B_n$ can be derived from coefficient of collective enhancement of level density K_{coll} :

$$\rho(U, J, \pi) = \rho_{\text{qp}}(U, J, \pi)K_{\text{coll}}(U, J, \pi). \quad (1)$$

In accordance with modern notions, K_{coll} determines [12] degree of increase in density of pure quasiparticle excitations $\rho_{\text{qp}}(U, J, \pi)$ in deformed nucleus due to its vibrations and rotation. One can accept in the first approach that, to a precision of small constant, it equals coefficient of vibrational enhancement of level density K_{vibr} .

On the whole, this coefficient is determined by change in entropy δS of a nucleus and redistribution of nuclear excitation energy δU between quasiparticles and phonons at nuclear temperature T :

$$K_{\text{vibr}} = \exp(\delta S - \delta U/T). \quad (2)$$

Now there is a possibility for unambiguous experimental determination [13] of breaking threshold E_N for the first and following Cooper pairs, value and shape of correlation functions δ_N of nucleon pair number N in heated nuclei. The main uncertainty of E_N is caused by the lack of experimental data on function $\delta_N = f(U)$, the secondary - by uncertainty of one-quasiparticle level density g in model [14]. So, three different model dependent approximations of level density in large set of nuclei ([13] and [15]) predict threshold E_2 for five-quasiparticle excitations which differs by a factor of 1.5-2.0.

In practice, we used the second variant of notions of the Cooper pair correlation function in heated nucleus [13] for estimation of the K_{vibr} value from approximation of the data [7, 8]. The values $\delta_1 = 1.02$ MeV, $g = 9.95$ MeV $^{-1}$ were used in calculation. Densities of three-quasiparticle levels (multiplied by the “best” $K_{\text{vibr}} = \text{const}$ value) calculated for the breaking threshold of the first Cooper pair of nucleons $E_1 = 0$ MeV are given in fig. 7. Its concrete values for minimal χ^2 are equal to 9.6 and 6.3 for ^{157}Gd and ^{159}Gd , respectively. The assumption on energy independence of K_{vibr} at low excitation energy is evidently unreal (see fig. 7).

Parameter $K_{\text{coll-1}}$ determined from comparison between calculated in this way density of three-quasiparticle excitations ($J = 1/2, 3/2$) and its most probable experimental value is compared with calculated value δ_1 in Fig. 7.

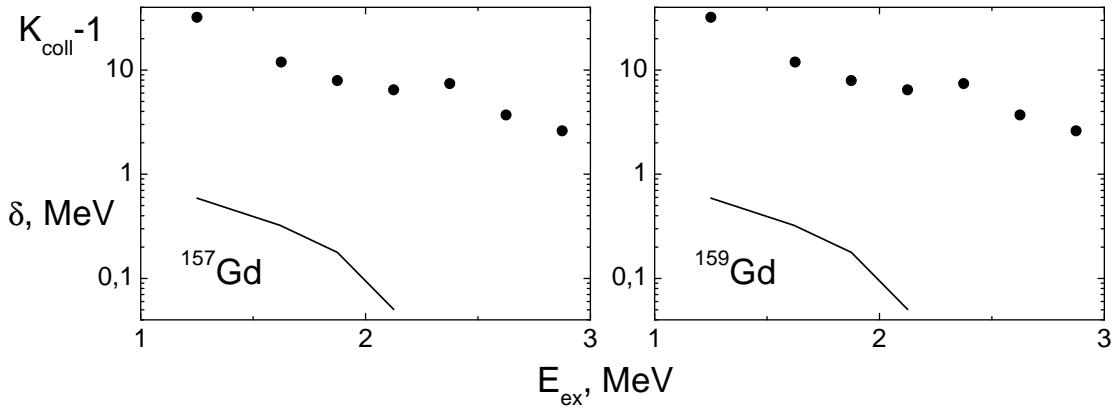


Fig. 7. Points show coefficient of collective enhancement of level density, curve represents the values of parameter δ_1 used in [13, 15] for calculation of partial density of three-quasiparticle levels.

In the excitation energy interval below ≈ 2 MeV is observed considerable correlation of this coefficient with the δ_1 value from [15] and from the second variant of analysis [13]. Decrease in correlation at higher excitation energy can be related to significant contribution of five-quasiparticle excitations in function $\rho_{qp}(U, J, \pi)$ and/or to smaller than it is adopted in [13, 15] rate of decrease of function δ_1 at $U > 1.8$ MeV.

Analysis of experimental data presented above in common with the data [1, 11] points to necessity of experimental search for neutron resonance structure in the energy interval E_n of about 1-2 MeV and more. Figure 6 show both general properties of radiative strength functions in isotopes under consideration and their evident difference in region below $\sim 0.5B_n$.

Analogous variations of energy dependence of radiative strength functions in both nuclei of neighboring elements and isotopes are also demonstrated by analysis of the two-step cascade intensities. If these changes are completely or to a great extent determined by difference in ratio between quasiparticle and phonon components of the neutron resonance wave functions then extrapolation of the regularity established in [15] for expected difference of breaking threshold of two neighboring Cooper pairs $E_{N+1} - E_N \approx 2\delta$ can be spread and into the region above B_n . Therefore, one can expect cyclic change in structure of excited resonances with period of about 2 MeV for heavy nuclei with $\delta \sim 1$ MeV.

Moreover, this effect in even-odd isotopes of rear earth elements can appear itself as change in ratio between intensities of the primary transitions with energy $E_1 = 3 - 4$ MeV and $E_1 > B_n - 1$ MeV.

The data presented allow the following conclusions:

1. Nuclei $^{157,159}\text{Gd}$, excited in the (\bar{n}, γ) reaction by the 2 keV and 24 keV neutrons demonstrate the same properties as those revealed earlier for about forty nuclei from the mass region $40 \leq A \leq 200$: step-wise structure in level density and local strengthening of radiative strength functions of the primary gamma-transitions to the levels of this structure.
2. Abrupt change in structure of levels in the excitation energy region about 1.0-1.5 MeV. It manifests itself in considerable increase of the $k(M1)/k(E1)$ values and in strong difference between their distribution and normal distribution of random gamma-transition

amplitudes.

3. These experimental ratios $k(M1)/k(E1)$ can be used for obtaining of simpler values of the E1- and M1-transition strength functions and data on the ratio between density of levels with different parity in the frameworks of methods [1, 11].

4. Main part of the primary gamma-transitions observed in the (\bar{n}, γ) reaction corresponds, probably, to excitation of levels with large and weakly fragmented phonon components of wave functions.

5. The data on the most probable N_γ values obtained for different intervals of neutron energy allow one to consider practically negligible dependence of the determined level density on E_n . Besides, influence of nuclear resonance structures results in considerable change in shape of the deviation distribution of the gamma-transition intensities from its mean value even in the narrow interval of nuclear excitation energy considered here.

5 Conclusion

Analysis of the available data on the primary gamma-transition intensities from (\bar{n}, γ) reaction in compound nuclei $^{157,159}\text{Gd}$ showed step-like structure in density of their levels and increase in radiative strength function of transitions to levels in region of this structure, at least, for primary dipole gamma-transitions. Id est, it confirmed main conclusions of [1, 11] and pointed to necessity to reveal and remove systematical experimental errors in alternative methods for determination of only level density [19] and simultaneous determination of all the parameters of cascade gamma-decay [20, 21]. The most important problems at experimental determination of level density and emission probability of reaction product become both correct accounting for influence of level structure on emission probability of nuclear evaporation and cascade gamma-quanta in investigations of nuclear reactions on accelerator beams and considerable decrease in systematical errors of experiment.

The best estimations of dispersion (parameter ν) of the random intensity fluctuations strongly differ from the mean values predicted in [22]). This allows one to assume that the wave functions of levels excited by primary transitions contain considerable components of weakly fragmented nuclear states which are more complicated than one- or three-quasiparticle states. Approximation of the obtained level density by model [14] confirms the fact of considerable (≈ 10 times) increase of level density due to excitations of mainly vibration type [15]. Comparison of the data presented in figs. 5,6 with those obtained from intensities of two-step cascades permits one to make preliminary conclusion that the abrupt change in structure of decaying neutron resonances, at least, in their energy interval ≈ 24 keV is not observed. There is no reason to wait principle change in the determined according to method [1, 11] level density and energy dependence of the primary transition radiative strength functions from resonance to resonance. Further decrease of errors of these nuclear parameters determined from intensities of the two-step gamma-cascades undoubtedly requires reliable estimation of function $k(E_\gamma, E_{ex})$ practically in all energy diapason of levels excited at thermal neutron capture.

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